NELSON SENIOR MATHS METHODS 12
FULLY WORKED SOLUTIONS

Chapter 2 Discrete random variables
Exercise 2.01 Discrete random variables

Concepts and techniques

1 **a** Continuous

b Discrete

c Discrete

d Continuous

e Discrete

f Continuous

g Discrete

h Continuous

2 **a** $X = \{1, 2, 3, 4, 5, 6\}$

b $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

c $X = \{0, 1, 2, 3\}$

d $X = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

e $X = \{x: 0 \leq x \leq 2\}$

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3 **a** Discrete, random

b Discrete, random

c Discrete, random

d Discrete, non-random (the number chosen by a person could be influenced by their personal preferences and is therefore not random)

e Continuous, random

4 D as height can be measured as accurately as you want

5 C as the number of traffic light stops is a countable number

6 B because you can stop at 0, 1, … up to 7 of the traffic lights

7 E $P(\text{2 different}) = \frac{{}^2C_1 \times {}^3C_1}{{}^5C_2} = \frac{2\times 3}{10} = \frac{3}{5}$

8 D $P(R\cap T) = P(R) \times P(T|R)$

$= 0.2 \times 0.5$

$P(R\cap T) = 0.1$

$P(R\cup T) = P(R) + P(T) - P(R\cap T)$

$= 0.2 + 0.6 - 0.1$

$P(R\cup T) = 0.7$

9 D $P(M \cup Q) = P(M) + P(Q) - P(M \cap Q)$

$0.7 = 0.4 + 0.5 - P(M \cap Q)$

$P(M \cap Q) = 0.2$

$\therefore$ M and Q are NOT mutually exclusive.

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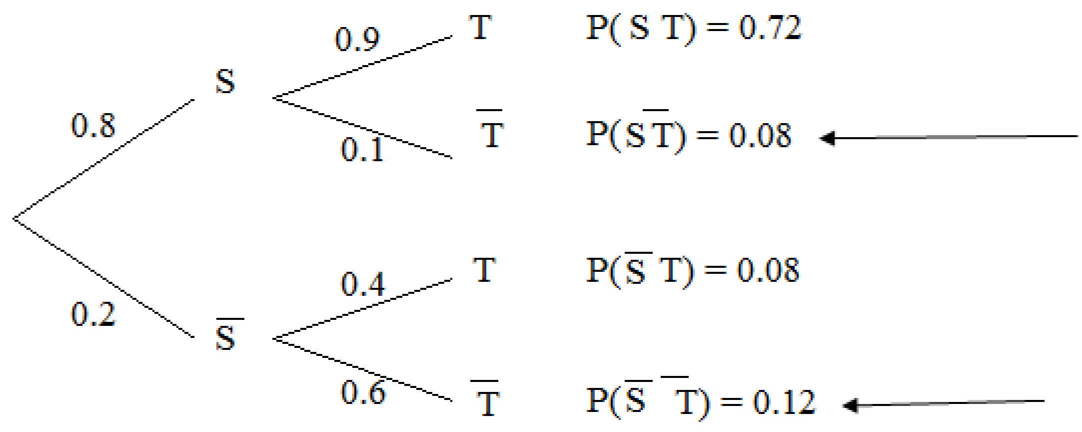
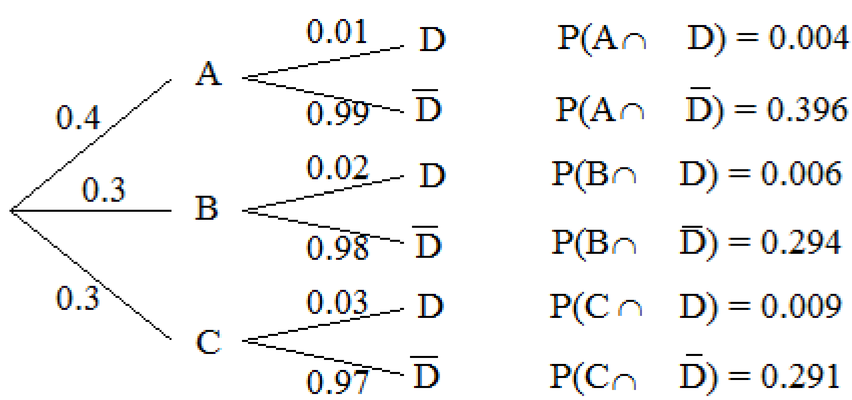
10 D S = ‘car starts’, T = ‘on time’



P(late) = 0.08 + 0.12 = 0.2

11



a P(not defective) = 0.396 + 0.294 + 0.291

= 0.981

b $P(B|D) = \dfrac{0.006}{1-0.981} = 0.316 = \dfrac{6}{19}$

12 Possible outcomes and sum of faces

1, 1	2, 1	3, 1	4, 1
1, 2	2, 2	3, 2	4, 2
1, 3	2, 3	3, 3	4, 3
1, 4	2, 4	3, 4	4, 4

2	3	4	5
3	4	5	6
4	5	6	7
5	6	7	8

$n(\mathrm{S}) = 16$

$(2, \frac{1}{16}), (3, \frac{1}{8}), (4, \frac{3}{16}), (5, \frac{1}{4}), (6, \frac{3}{16}), (7, \frac{1}{8}), (8, \frac{1}{16})$

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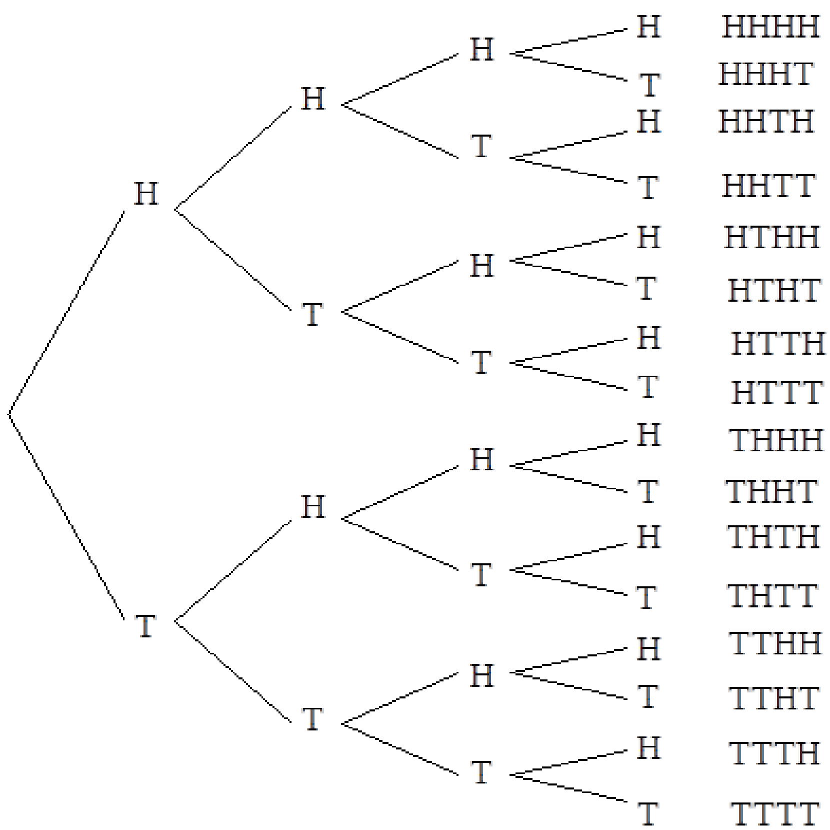
Reasoning and communication

13 Successive heads means *heads in a row*



a $P(\text{H} = 3) = \dfrac{2}{16} = 0.125$

b $P(\text{H} > 3) = \dfrac{1}{16} = 0.0625$

c $P(\text{H} < 3) = 1 - P(\text{H} = 3 \text{ or } \text{H} = 4) = 1 - \dfrac{3}{16} = \dfrac{13}{16} = 0.8125$

d $P(\text{H} = 5) = 0$

e $P(\text{H} \geq 2) = 1 - (\text{H} = 0 \text{ or } \text{H} = 1) = 1 - \dfrac{8}{16} = \dfrac{8}{16} = 0.5$

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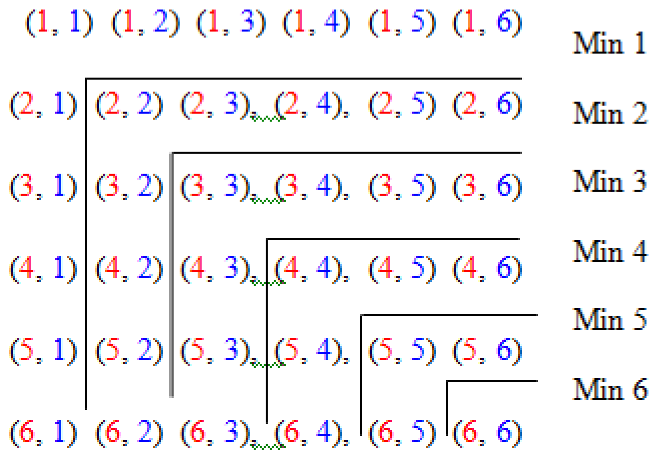
f {0, 1, 2, 3, 4},

$p(0) = 0.0625$, $p(1) = 0.4375$, $p(2) = 0.3125$, $p(3) = 0.125$, $p(4) = 0.0625$

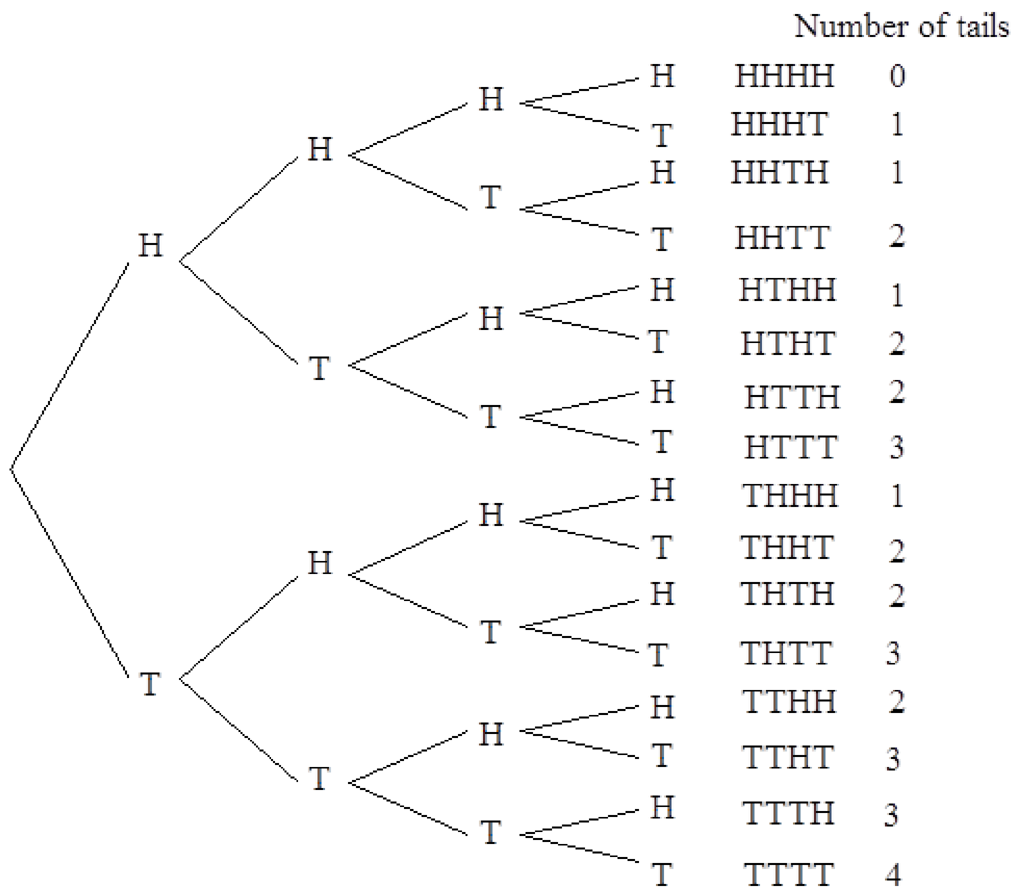
14 $M \in \{1, 2, 3, 4, 5, 6\}$

Two dice



$P(M = m) = (1, \frac{11}{36}), (2, \frac{1}{4}), (3, \frac{7}{36}), (4, \frac{5}{36}), (5, \frac{1}{12}), (6, \frac{1}{36})$

15



a $T \in \{0, 1, 2, 3, 4\}$

$P(T = t) = (0, \frac{1}{16}), (1, \frac{1}{4}), (2, \frac{3}{8}), (3, \frac{1}{4}), (4, \frac{1}{16})$

b $P(T > 2) = P(\text{T} = 3 \text{ or T} = 4) = \frac{1+4}{16} = \frac{5}{16}$

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Exercise 2.02 Discrete probability distributions

Concepts and techniques

1 A

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)

(2, 1) **(2, 2)** (2, 3) (2, 4) (2, 5) (2, 6)

(3, 1) (3, 2) **(3, 3)** (3, 4) (3, 5) (3, 6)

(4, 1) (4, 2) (4, 3) **(4, 4)** (4, 5) (4, 6)

(5, 1) (5, 2) (5, 3) (5, 4) **(5, 5)** **(5, 6)**

(6, 1) (6, 2) (6, 3) (6, 4) **(6, 5)** **(6, 6)**

There are 5 doubles that do not add to more than 10. $\frac{5}{36}$

There are 2 pairs that add to more than ten but are not a double. $\frac{2}{36}=\frac{1}{18}$

There is 1 double that adds to more than 10. $\frac{1}{36}$

There are 28 other possible combinations. $\frac{28}{36}=\frac{7}{9}$

2 D $p(x)=\frac{(x-2)^3}{35}$

$p(1)=\frac{-1}{35}$

It is not a probability function because $p(1) < 0$.

3 B

x	2	5	8	12
$P(X = x)$	$2m$	$5m$	$6m$	$3m$

$2m + 5m + 6m + 3m = 1$

$16m = 1 \Rightarrow m = \frac{1}{16} = 0.0625$

4 E

For the discrete probability distribution shown below.

x	3	5	8	9	11
$P(X = x)$	0.12	0.25	0.33	0.21	0.09

$P(x \leq 8) = 0.12 + 0.25 + 0.33 = 0.7$

5 B The sample space for tossing three coins is

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

$X = \{0, 1, 2, 3\}$

x	0	1	2	3
$p(x)$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

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6 B

x	0	1	2	3	4
$P(X = x)$	0.15	0.25	n	0.35	0.05

$0.15 + 0.25 + n + 0.35 + 0.05 = 1$

$n = 0.2$

7 **a**

x	$p(x)$
1	0.3
2	0.2
3	0.4

$\sum p(x) = 0.9$ instead of 1 so it could NOT represent a probability distribution.

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b

m	$p(m)$
−3	0.1
−1	0.2
1	0.3
3	0.4

$\sum p(x) = 1$ so could represent a probability distribution.

c

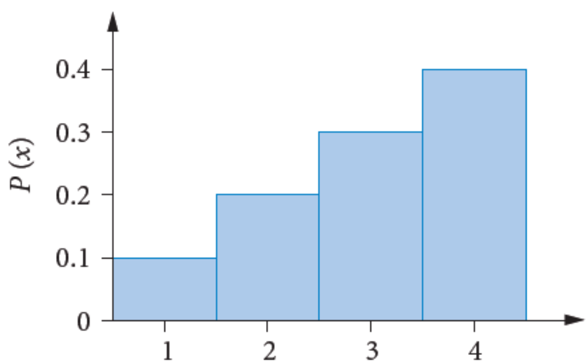
t	$p(t)$
0	0.4
2	−0.1
6	0.2
8	0.5

$p(2) < 0$ so could NOT represent a probability distribution.

8 **a**

x	$p(x)$
1	0.1
2	0.2
3	0.3
4	0.4

b



c 0.1+ 0.2 + 0.3 + 0.4 = 1, which satisfies the condition to be a probability distribution

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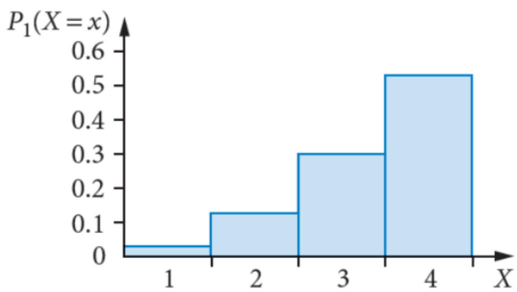
9 $p_1 = \frac{x^2}{30}$ and $p_2 = \frac{5x^3}{12} - \frac{x^4}{24} - \frac{3}{4} - \frac{35x^2}{24} + \frac{25x}{12}$ for x = 1, 2, 3, 4

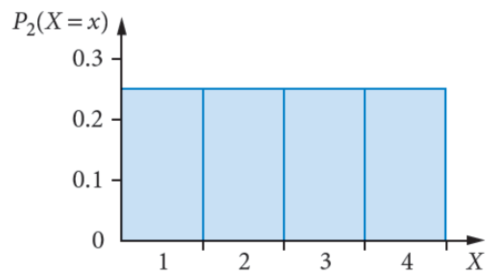
a

x	1	2	3	4
$p_1(x)$	$\frac{1}{30}$	$\frac{4}{30}$	$\frac{9}{30}$	$\frac{16}{30}$
$p_2(x)$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

b $\frac{1}{30}+\frac{4}{30}+\frac{9}{30}+\frac{16}{30}=1$, $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=1$, all positive.

c





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Reasoning and communication

10

Two tetrahedral dice	Sum of faces F
(1, 1) (1, 2) (1, 3) (1, 4)	(2) (3) (4) (5)
(2, 1) (2, 2) (2, 3) (2, 4)	(3) (4) (5) (6)
(3, 1) (3, 2) (3, 3) (3, 4)	(4) (5) (6) (7)
(4, 1) (4, 2) (4, 3) (4, 4)	(5) (6) (7) (8)

f	2	3	4	5	6	7	8
$P(F = f)$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{3}{16}$	$\frac{1}{4}$	$\frac{3}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

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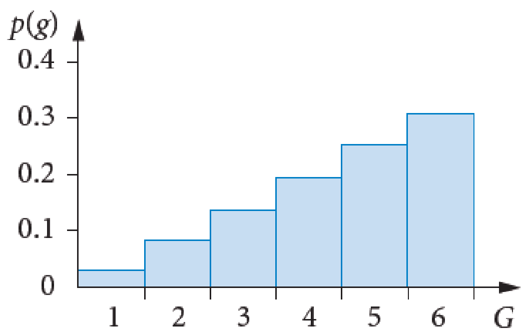
11 Two dice | Maximum on upper faces

Two dice	Maximum on upper faces
(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)	(1) (2) (3) (4) (5) (6)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)	(2) (2) (3) (4) (5) (6)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)	(3) (3) (3) (4) (5) (6)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)	(4) (4) (4) (4) (5) (6)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)	(5) (5) (5) (5) (5) (6)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)	(6) (6) (6) (6) (6) (6)

a

g	1	2	3	4	5	6
$P(G = g)$	$\frac{1}{36}$	$\frac{1}{12}$	$\frac{5}{36}$	$\frac{7}{36}$	$\frac{1}{4}$	$\frac{11}{36}$

b



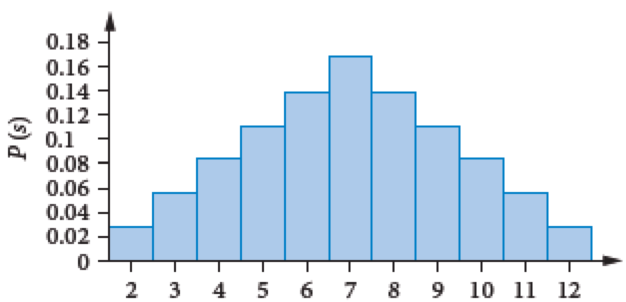
12 Two six-sided dice | Sum of faces (S)

Two six-sided dice	Sum of faces (S)
(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)	(2) (3) (4) (5) (6) (7)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)	(3) (4) (5) (6) (7) (8)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)	(4) (5) (6) (7) (8) (9)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)	(5) (6) (7) (8) (9) (10)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)	(6) (7) (8) (9) (10) (11)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (6, 6)	(7) (8) (9) (10) (11) (12)

a

x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

b



c $$\sum p(x_i) = \frac{1}{36}+\frac{2}{36}+\frac{3}{36}+\frac{4}{36}+\frac{5}{36}+\frac{6}{36}+\frac{5}{36}+\frac{4}{36}+\frac{3}{36}+\frac{2}{36}+\frac{1}{36}=\frac{36}{36}=1$$

2 3 4 5 6 7 8 9 10 11 12

Therefore it is a probability distribution.

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13 $P(\text{H}) = \frac{2}{3}$ and $P(\text{T}) = \frac{1}{3}$

Let X be the random variable representing the largest number of successive heads that occur.

HHH, HHT, HTH, THH, HTT, THT, TTH, TTT

$P(\text{HHH}) = \left(\frac{2}{3}\right)^3 = \frac{8}{27}$

$P(\text{two heads}) = 3\times\left(\frac{2}{3}\right)^2\times\left(\frac{1}{3}\right) = \frac{12}{27}$

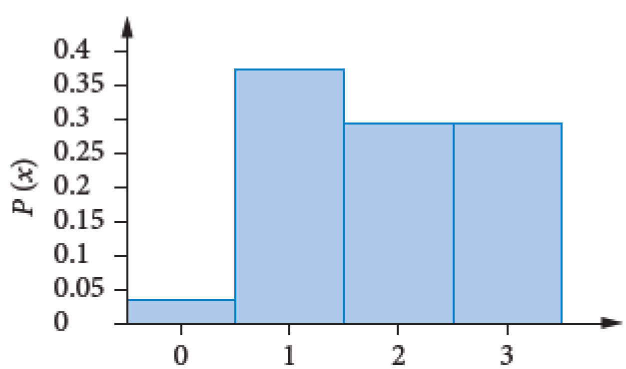
$P(\text{one head}) = 3\times\left(\frac{2}{3}\right)\times\left(\frac{1}{3}\right)^2 = \frac{6}{27}$

$P(\text{no heads}) = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$

a

x	0	1	2	3
***p*(*x*)**	$\frac{1}{27}$	$\frac{10}{27}$	$\frac{8}{27}$	$\frac{8}{27}$

b



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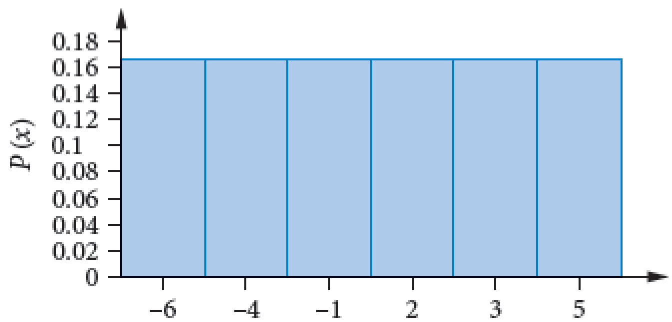
14 Prime numbers = {2, 3, 5}, Non-prime numbers = {1, 4, 6}

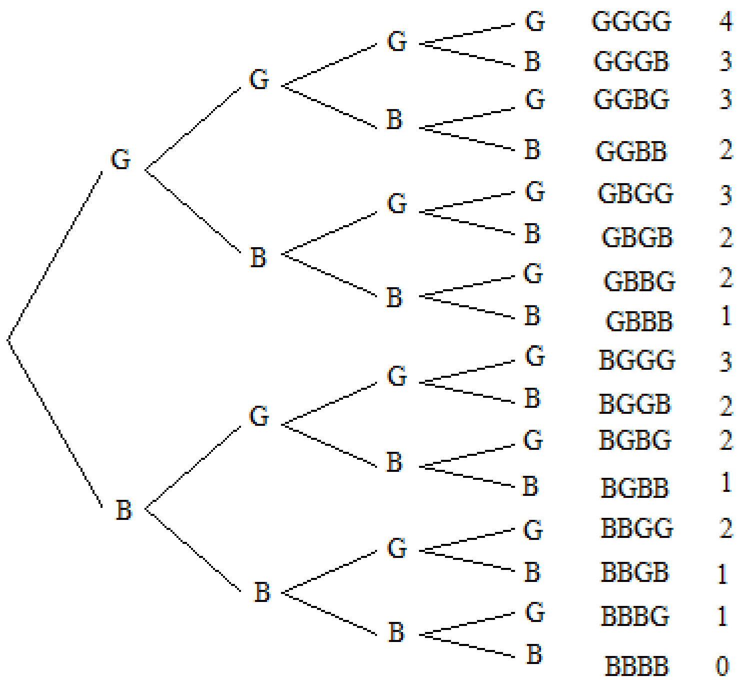
a

x	−6	−4	−1	2	3	5
$p(x)$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$

b



15 Number of girls



x	0	1	2	3	4
***p*(*x*)**	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

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16 $n = 6 \times 6 \times 6 = 216$

n(triples) = 6 and win \$20

n(exactly two the same) = ${}^6C_1 \times {}^5C_1 \times 3 = 90$ and win \$5

n(all different numbers) = 216 – 6 – 90 = 120

		triple	double	all different
Winnings	***x***	20	5	–5
	***p*(*x*)**	$\frac{6}{216}=\frac{1}{36}$	$\frac{90}{216}=\frac{5}{12}$	$\frac{120}{216}=\frac{5}{9}$

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Exercise 2.03 Estimating probabilities

Concepts and techniques

1 $\frac{389}{1344} = 0.289$

2 **a** $\frac{233}{1309} = 0.178$

b $\frac{53}{286} = 0.185$

3 $\frac{16}{40} = 0.4$

4 **a** $\frac{23800}{85\,200} = 0.279$

b $\frac{2\,300}{23800} = 0.097$

5 **a** $\frac{94100}{292\,400} = 0.322$

b $\frac{10\,100}{94\,100} = 0.107$

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Exercise 2.04 Uniform discrete probability distributions

Concepts and techniques

1 A There are 12 sides, only one of which is an 8 so $\frac{1}{12}$.

2 B There are 5 equally likely outcomes, so $\frac{1}{5}$.

3 **a** $N = \{1, 2, 3, 4\}$ All equally likely, so uniform.

b $X = \{0, 1, 2, 3, 4, \ldots, 9\}$ All equally likely, so uniform.

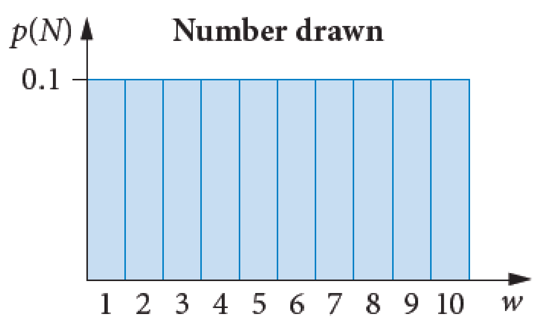
c $S = \{0, 1, 2, 3, 4, \ldots, 9\}$ All equally likely, so uniform.

d $D = \{1, 2, 3, 4, \ldots, 20\}$ Non-uniform. Probability of a particular disc on draw one is $\frac{1}{20}$ but on the second draw $\frac{1}{19}$ or 0.

e $M = \{1, 2, 3, 4, 5, 6\}$ All equally likely, so uniform.

f $T = (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ Not equally likely, so non-uniform

4



All numbers are equally likely to be drawn, so uniform distribution.

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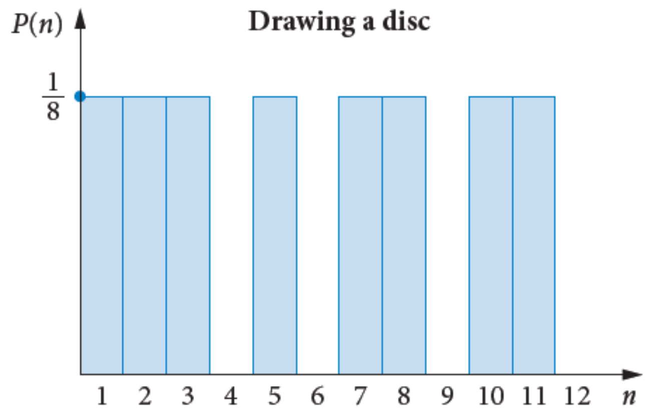
5 **a**



b $P(n \leq 4) = \frac{3}{8}$

c $P(n \neq 8) = \frac{7}{8}$

d $P(n \text{ is even}) = \frac{3}{8}$

e $P(n \text{ is not even}) = \frac{5}{8}$

f $P(2 \leq n \leq 8) = \frac{5}{8}$

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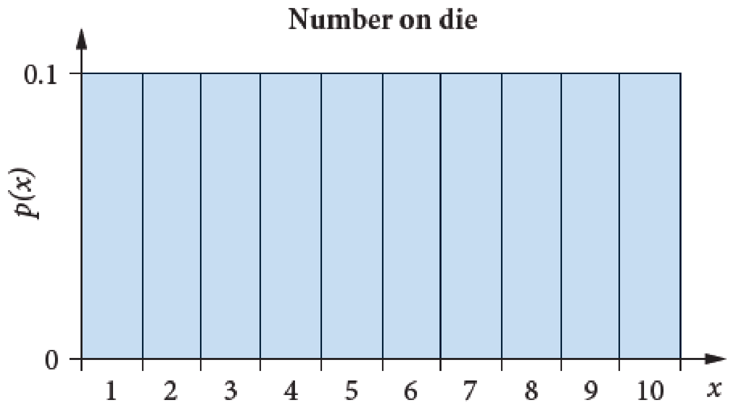
6 **a**



b $P(x \geq 2) = 0.9$

c $P(x < 4) = 0.3$

d $P(2 \leq x \leq 5) = 0.4$

7 $S = \{1, 2, 3, \ldots, 30\}$

a $P(s = 17) = \frac{1}{30}$

b $P(s \neq 17) = \frac{29}{30}$

c $P(5 \leq s \leq 22) = \frac{3}{5}$

d $P(s > 17) = \frac{13}{30}$

e $P(s \leq 17) = \frac{17}{30}$

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Reasoning and communication

8 **a** $P(t > 8 \text{ min}) = \dfrac{240}{571} \approx 0.4203$

b $P(t < 5 \text{ min}) = \dfrac{150}{571} \approx 0.2627$

c $P(5 \text{ min} \le t \le 9 \text{ min}) = \dfrac{241}{571} \approx 0.4221$

d $P(7 \text{ min} \le t \le 15 \text{ min}) = \dfrac{301}{571} \approx 0.5271$

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Exercise 2.05 The hypergeometric distribution

Concepts and techniques

1 A As many as 5 can be selected in the sample of size 5.

2 D The probability of success in each trial is constant is not true because if one is already chosen and not replaced, it reduces the probability of it being selected again

3 B As 10 is the most that can be chosen.

For Questions **4** and **5**. $P(X = 3) = \dfrac{\binom{12}{3}\binom{38}{2}}{\binom{50}{5}}$

4 B 5, as 3 of one type are being selected and 2 of the other type are being selected.

5 D X is the variable 'success' and we are choosing 3 from 12.

6 **a** $N = 52$, $k = 13$, $n = 4$, $X \in \{0, 1, 2, 3, 4\}$

b $N = 6$, $k = 2$, $n = 3$, $X \in \{0, 1, 2\}$

c $N = 50$, $k = 9$, $n = 10$, $X \in \{0, 1, 2, \ldots, 9\}$

d $N = 200$, $k = 20$, $n = 40$, $X \in \{0, 1, 2, \ldots, 20\}$

e $N = 95$, $k = 35$, $n = 12$, $X \in \{0, 1, 2, \ldots, 12\}$

7 **a** $P(x = 2) = \dfrac{{}^{39}C_2 \times {}^{13}C_2}{{}^{52}C_4} = 0.2135$

b $P(x = 1) = \dfrac{{}^{4}C_2 \times {}^{2}C_1}{{}^{6}C_3} = 0.6$

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c $P(x = 4) = \dfrac{{}^{41}C_6 \times {}^{9}C_4}{{}^{50}C_{10}} = 0.0552$

d $P(x = 10) = \dfrac{{}^{180}C_{30} \times {}^{20}C_{10}}{{}^{200}C_{40}} = 0.0012$

e $P(x = 3) = \dfrac{{}^{60}C_9 \times {}^{35}C_3}{{}^{95}C_{12}} = 0.1770$

8 **a** $N = 120$, $n = 20$, $k = 30$, $x = 7$

$P(x = 7) = \dfrac{{}^{90}C_{13} \times {}^{30}C_7}{{}^{120}C_{20}} = 0.1136$

b $N = 50$, $n = 15$, $k = 10$, $x = 3$

$P(x = 3) = \dfrac{{}^{40}C_{12} \times {}^{10}C_3}{{}^{50}C_{15}} = 0.2979$

c $N = 200$, $n = 50$, $k = 40$, $x = 9$

$P(x = 9) = \dfrac{{}^{160}C_{41} \times {}^{40}C_9}{{}^{200}C_{50}} = 0.1523$

d $N = 30$, $n = 8$, $k = 5$, $x = 3$

$P(x = 3) = \dfrac{{}^{25}C_5 \times {}^{5}C_3}{{}^{30}C_8} = 0.0908$

e $N = 150$, $n = 25$, $k = 40$, $x = 11$

$P(x = 11) = \dfrac{{}^{110}C_{14} \times {}^{40}C_{11}}{{}^{150}C_{25}} = 0.0217$

Reasoning and communication

9 $P(x = 2 \text{ spades}) = \dfrac{{}^{39}C_4 \times {}^{13}C_2}{{}^{52}C_6} = 0.3151$

10 **a** $P(\text{three will be defective}) = \dfrac{{}^{85}C_{16} \times {}^{15}C_3}{{}^{100}C_{19}} = 0.2705$

b P(at least three will be defective) = 1 – P(0) – P(1) – P(2)

= 1 – 0.0321… – 0.1367… – 0.2534…

= 0.5777

11 **a** 200 = 188 + 12, N = 200, k = 12, n = 20

$P(\text{exactly three are colourblind}) = \dfrac{{}^{188}C_{17} \times {}^{12}C_3}{{}^{200}C_{20}} = 0.0833$

b P(at least one is colourblind) = 1 – P(0)

$= 1 - \dfrac{{}^{188}C_{20} \times {}^{12}C_0}{{}^{200}C_{20}} = 1 - 0.2718 = 0.7282$

12 **a** $P(\text{a single entry will win the jackpot}) = \dfrac{{}^{39}C_0 \times {}^{6}C_6}{{}^{45}C_6} = 0.000\ 000\ 1228$

b P(a single entry will win a prize) = P(3) + P(4) + P(5) + P(6)

$= \dfrac{{}^{39}C_3 \times {}^{6}C_3}{{}^{45}C_6} + \dfrac{{}^{39}C_2 \times {}^{6}C_4}{{}^{45}C_6} + \dfrac{{}^{39}C_1 \times {}^{6}C_5}{{}^{45}C_6} + \dfrac{{}^{39}C_0 \times {}^{6}C_6}{{}^{45}C_6}$

= 0.022 441 + 0.001 364 + 0.000 029 + 0.000 000 1

= 0.0238

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13 **a** P(no unusable components are found) = $\frac{^{238}C_{20}}{^{250}C_{20}} = 0.3590$

b P(three unusable components are found) = $\frac{^{238}C_{17} \times {}^{12}C_3}{^{250}C_{20}} = 0.0507$

c P(the order will be rejected) = $P(x \geq 4)$

$= 1 - P(0) - P(1) - P(2) - P(3)$

$= 1 - 0.3598 - 0.3935 - 0.1869 - 0.0507$

$= 0.0098$

14 P(the trial will result in a hung jury) = $P(x \geq 1)$

$= 1 - P(0)$

$= 1 - \frac{^{47}C_{12} \times {}^{3}C_0}{^{50}C_{12}}$

$= 1 - 0.4304$

$= 0.5696$

Exercise 2.06 Expected value

Concepts and techniques

1 C 22.9, as the average of the scores is 22.9

2 D As $1.1 = 0 \times 0.2 + 1 \times 0.5 + 2 \times 0.3$

3 B As $1.20 = 0 \times 0.2401 + 1 \times 0.4116 + 2 \times 0.2646 + 3 \times 0.0756 + 4 \times 0.0081$

4 $E(X) = 0 \times 0.5 + 5 \times 0.1 + 10 \times 0.2 + 15 \times 0.2 = 5.5$

5 **a** $E(X) = 2 \times \frac{1}{3} + 3 \times \frac{1}{2} + 11 \times \frac{1}{6}$

$= 4$

b $E(X) = -5 \times \frac{1}{4} - 4 \times \frac{1}{8} + 1 \times \frac{1}{2} + 2 \times \frac{1}{8}$

$= -1$

c $E(X) = 1 \times 0.4 + 3 \times 0.1 + 4 \times 0.2 + 5 \times 0.3$

$= 3$

6 $E(X) = 0 \times 0.1 + 1 \times 0.2 + 2 \times 0.3 + 5 \times 0.4$

$= 2.8$

7 **a** F: $E(X) = 57.5$

b M: $E(X) = 0.3 \times 57.5 = 17.25$

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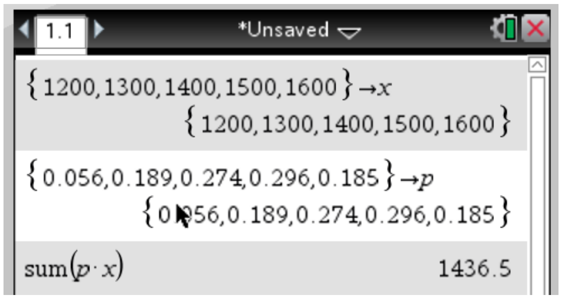
8 a

TI-Nspire CAS



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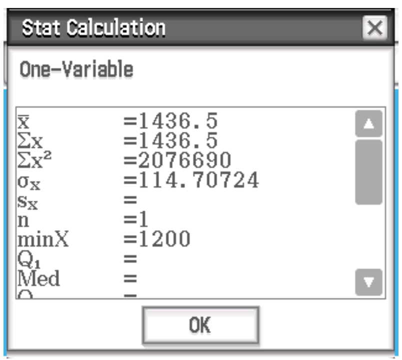

$E(X) = 1200 \times 0.056 + 1300 \times 0.189 + 1400 \times 0.274 + 1500 \times 0.296$

$+ 1600 \times 0.185$

$= 1437$

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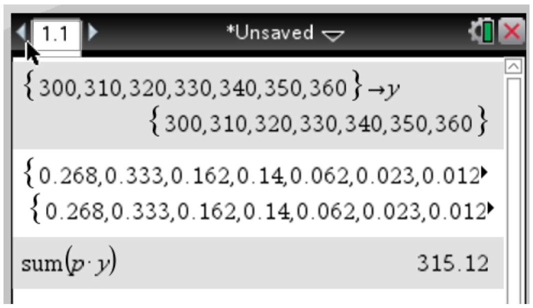
b

TI-Nspire CAS



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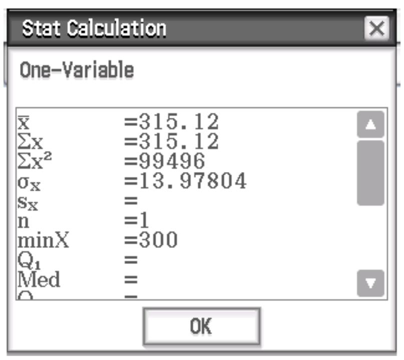

$E(X) = 300 \times 0.268 + 310 \times 0.333 + 320 \times 0.162 + 330 \times 0.14$

$+ 340 \times 0.062 + 350 \times 0.023 + 360 \times 0.012$

$= 315$

9 $E(X) = 1 \times \frac{1}{12} + 2 \times \frac{5}{12} + 3 \times \frac{1}{3} + 4 \times \frac{1}{6}$

$= 2\frac{7}{12}$

Reasoning and communication

10 $X = \{0, 1, 2, 3\}$

X	0	1	2	3
***P*(*x*)**	$\frac{1}{8}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$

a $E(X) = 0 \times \frac{1}{8} + 1 \times \frac{1}{2} + 2 \times \frac{1}{4} + 3 \times \frac{1}{8} = 1.375$

b $P(\text{H}) = \frac{2}{3}$

X	0	1	2	3
***P*(*x*)**	$\frac{1}{27}$	$\frac{10}{27}$	$\frac{8}{27}$	$\frac{8}{27}$

$E(X) = 0 \times \frac{1}{27} + 1 \times \frac{10}{27} + 2 \times \frac{8}{27} + 3 \times \frac{8}{27} = 1.852$

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11 **a** 1 1 1 1 1 1

1 2 2 2 2 2

1 2 3 3 3 3

1 2 3 4 4 4

1 2 3 4 5 5

1 2 3 4 5 6

S	1	2	3	4	5	6
$P(s)$	$\frac{11}{36}$	$\frac{9}{36}=\frac{1}{4}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}=\frac{1}{12}$	$\frac{1}{36}$

b $E(S) = 2.528$

12 **a**

(1, 2) (1, 3) (1, 4) (1, 5)   (3) (4) (5) (6)

(2, 1) (2, 3) (2, 4) (2, 5)   (3) (5) (6) (7)

(3, 1) (3, 2) (3, 4) (3, 5)   (4) (5) (7) (8)

(4, 1) (4, 2) (4, 3) (4, 5)   (5) (6) (7) (9)

(5, 1) (5, 2) (5, 3) (5, 4)   (6) (7) (8) (9)

S	3	4	5	6	7	8	9
P(s)	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{10}$	$\frac{1}{10}$

b $E(S) = 3 \times \frac{1}{10} + 4 \times \frac{1}{10} + 5 \times \frac{1}{5} + 6 \times \frac{1}{5} + 7 \times \frac{1}{5} + 8 \times \frac{1}{10} + 9 \times \frac{1}{10} = 6$

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13 Two dice

Two dice						Differences
(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)	0 1 2 3 4 5
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)	1 0 1 2 3 4
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)	2 1 0 1 2 3
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)	3 2 1 0 1 2
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)	4 3 2 1 0 1
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)	5 4 3 2 1 0

$P(\text{the numbers differ by three or more}) = \frac{12}{36} = \frac{1}{3}$

$P(\text{the numbers differ by one or two}) = \frac{18}{36} = \frac{1}{2}$

$P(\text{the numbers are equal}) = \frac{6}{36} = \frac{1}{6}$

***X* (number of places moved)**	1	2	4
***P*(*X* = *x*)**	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{1}{6}$

$E(X) = 1 \times \frac{1}{3} + 2 \times \frac{1}{2} + 4 \times \frac{1}{6} = 2$

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14

x	1	2	3	4	5
$P(X = x)$	$8k$	$5k$	$4k$	$2k$	k

a $8k + 5k + 4k + 2k + k = 1$

$20k = 1$

$k = \frac{1}{20} = 0.05$

b The probability of 1 and 2 are relatively high, so expect $E(X)$ to be less than 3.

c $E(X) = 1 \times \frac{8}{20} + 2 \times \frac{5}{20} + 3 \times \frac{4}{20} + 4 \times \frac{2}{20} + 5 \times \frac{1}{20} = \frac{43}{20} = 2.15$

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15

w	2	3	5	8	12
$P(W = w)$	$\frac{1}{8}$	$\frac{1}{3}$	$\frac{1}{4}$	x	y

$$\frac{1}{8}+\frac{1}{3}+\frac{1}{4}+x+y=1$$

$$\frac{17}{24}+x+y=1$$

$$24x+24y=7 \qquad [1]$$

$E(X) = 5\frac{2}{3}$

so $2\times\frac{1}{8}+3\times\frac{1}{3}+5\times\frac{1}{4}+8x+12y=5\frac{2}{3}$

$2\frac{1}{2}+8x+12y=5\frac{2}{3}$

$8x + 12y = \frac{19}{6}$

$48x + 72y = 19$

$48x + 48y = 14 \qquad 2 \times [1]$

$24y = 5$

$y = \frac{5}{24}$

$$\frac{17}{24}+x+y=1$$

$$\frac{17}{24}+x+\frac{5}{24}=1$$

$$x=\frac{2}{24}$$

$$x=\frac{1}{12}, \quad y=\frac{5}{24}$$

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16 Outcomes : {H, TH, TTH, TTTH, TTTTH, TTTTT}

X = {1, 2, 3, 4, 5}

x	1	2	3	4	5
$P(X = x)$	0.5	0.25	0.125	0.0625	0.0625

$E(X) = 1 \times 0.5 + 2 \times 0.25 + 3 \times 0.125 + 4 \times 0.0625 + 5 \times 0.0625$

$= 1.9$

17 $N = 6$, $k = 2$, $n = 3$

x	0	1	2
$P(X = x)$	$\frac{^4C_3}{^6C_3} = \frac{1}{5}$	$\frac{^4C_2 \times {}^2C_1}{^6C_3} = \frac{3}{5}$	$\frac{^4C_1 \times {}^2C_2}{^6C_3} = \frac{1}{5}$

$E(X) = 0 \times \frac{1}{5} + 1 \times \frac{3}{5} + 2 \times \frac{1}{5} = 1$

Exercise 2.07 Variance and standard deviation

Concepts and techniques

The variance of Z is:

1 A 1.0 $\sigma_z^2 = E(Z^2) - \mu^2$, where $\mu = E(Z)$

$\mu = E(Z) = 1 \times 0.1 + 2 \times 0.2 + 3 \times 0.3 + 4 \times 0.4$

$\mu = 3$

$E(Z^2) = 1^2 \times 0.1 + 2^2 \times 0.2 + 3^2 \times 0.3 + 4^2 \times 0.4$

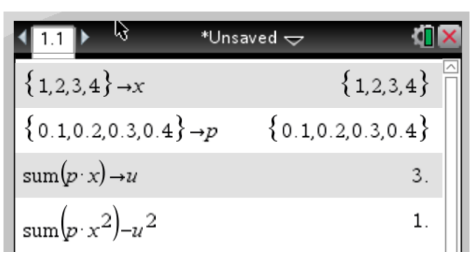
$= 0.1 + 0.8 + 2.7 + 6.4$

$= 10$

$\sigma_z^2 = E(Z^2) - \mu^2$

$= 10 - 9 = 1$

TI-Nspire CAS



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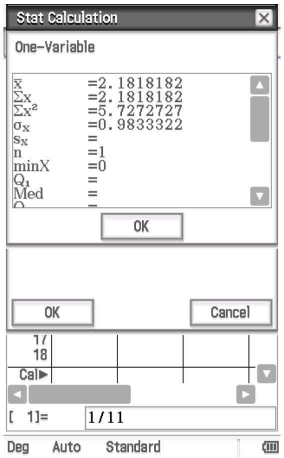

2 A $\sigma_y^2 = E(Y^2) - \mu^2$, where $\mu = E(Y)$

$\mu = E(Y) = 100 \times 0.2 + 200 \times 0.4 + 300 \times 0.3 + 400 \times 0.1$

$\mu = 230$

$E(Y^2) = 100^2 \times 0.2 + 200^2 \times 0.4 + 300^2 \times 0.3 + 400^2 \times 0.1$

$= 61\,000$

$\sigma_y^2 = E(Y^2) - \mu^2$

$= 61\,000 - 230^2 = 8100$

$\sigma_y = 90$

The standard deviation of Y is 90.

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3 A $a + 2a + 3a + 4a = 1 \Rightarrow a = 0.1$

$\sigma_m^2 = E(M^2) - \mu^2$, where $\mu = E(M)$

$\mu = E(M) = 1 \times 0.1 + 2 \times 0.2 + 3 \times 0.3 + 4 \times 0.4$

$\mu = 3$

$E(M^2) = 1^2 \times 0.1 + 2^2 \times 0.2 + 3^2 \times 0.3 + 4^2 \times 0.4$

$= 10$

$\sigma_m^2 = E(M^2) - \mu^2$

$= 10 - (3)^2 = 1$

$\sigma_m = 1$

The standard deviation of M is 1.

4 $\sigma_w^2 = E(W^2) - \mu^2$, where $\mu = E(W)$

$\mu = E(W) = 1 \times 0.1 + 2 \times 0.3 + 4 \times 0.4 + 6 \times 0.2$

$= 3.4$

$E(W^2) = 0^2 \times 0.1 + 2^2 \times 0.3 + 4^2 \times 0.4 + 6^2 \times 0.2$

$= 14$

$\sigma_w^2 = E(W^2) - \mu^2$

$= 14 - 3.4^2 = 3.24$

$\sigma_w = \sqrt{3.24} = 1.8$

The variance of W is 3.24 and the standard deviation of W is 1.8.

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5 $\sigma_x^2 = E(X^2) - \mu^2$

$= 280 - 14^2$

$= 84$

6 $\sigma_y^2 = E(Y^2) - \mu^2$

$= 587 - 23^2$

$= 58$

$\sigma_y \approx 7.62$

The standard deviation is 7.62.

7 **a** $n(X) = 20$

x	3	4	6	7	8	9
$p(x)$	0.05	0.05	0.15	0.45	0.2	0.1

b $E(X) = 3 \times 0.05 + 4 \times 0.05 + 6 \times 0.15 + 7 \times 0.45 + 8 \times 0.2 + 9 \times 0.1$

$= 6.9$

c $E(W^2) = 3^2 \times 0.05 + 4^2 \times 0.05 + 6^2 \times 0.15 + 7^2 \times 0.45 + 8^2 \times 0.2 + 9^2 \times 0.1$

$= 49.6$

$\sigma_w^2 = E(W^2) - \mu^2$

$= 49.6 - 6.9^2 = 1.99$

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8 **a**

n	0	1	2	3	4	5	6	7	8	9
***p*(*n*)**	$\frac{1}{30}$	$\frac{1}{30}$	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{2}{15}$	$\frac{1}{10}$	$\frac{1}{10}$	$\frac{1}{15}$

b $E(N) = 0 \times \frac{1}{30} + 1 \times \frac{1}{30} + 2 \times \frac{1}{15} + 3 \times \frac{1}{10} + 4 \times \frac{1}{5} + 5 \times \frac{1}{6} + 6 \times \frac{2}{15} + 7 \times \frac{1}{10}$

$+ 8 \times \frac{1}{10} + 9 \times \frac{1}{15}$

$= 5$

c $E(N^2) = 0^2 \times \frac{1}{30} + 1^2 \times \frac{1}{30} + 2^2 \times \frac{1}{15} + 3^2 \times \frac{1}{10} + 4^2 \times \frac{1}{5} + 5^2 \times \frac{1}{6} + 6^2 \times \frac{2}{15}$

$+ 7^2 \times \frac{1}{10} + 8^2 \times \frac{1}{10} + 9^2 \times \frac{1}{15}$

$= 30.07$

$\sigma_n^2 = E(N^2) - \mu^2$

$= 30.07 - 5^2 = 5.067$

d $\sigma_n = 2.25$

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9 **a** $E(N) = -1 \times 0.1 + 0 \times 0.3 + 1 \times 0.1 + 2 \times 0.2 + 3 \times 0.3$

$= 1.3$

$E(N^2) = (-1)^2 \times 0.1 + 0^2 \times 0.3 + 1^2 \times 0.1 + 2^2 \times 0.2 + 3^2 \times 0.3$

$= 3.7$

$\sigma_n^2 = E(N^2) - \mu^2$

$= 3.7 - 1.3^2 = 2.01$

$\sigma_n = 1.42$

b $E(R) = 11 \times 0.12 + 12 \times 0.18 + 13 \times 0.21 + 14 \times 0.19 + 15 \times 0.16$

$+ 16 \times 0.11 + 17 \times 0.03$

$= 13.54$

$E(R^2) = 11^2 \times 0.12 + 12^2 \times 0.18 + 13^2 \times 0.21 + 14^2 \times 0.19 + 15^2 \times 0.16$

$+ 16^2 \times 0.11 + 17^2 \times 0.03$

$= 186$

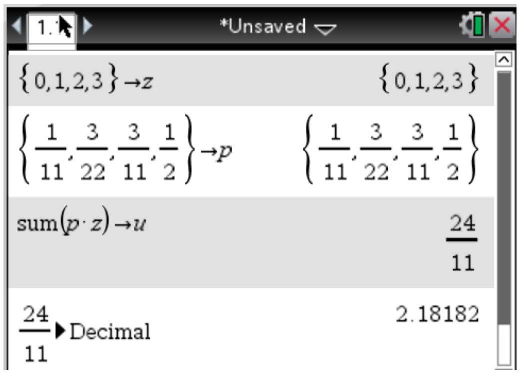
$\sigma_r^2 = E(R^2) - \mu^2$

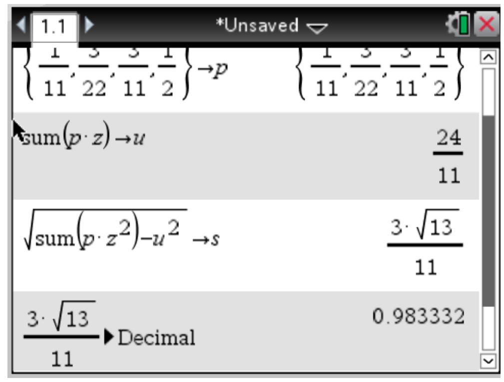
$= 186 - 13.54^2 = 2.67$

$\sigma_r = 1.63$

10

TI-Nspire CAS





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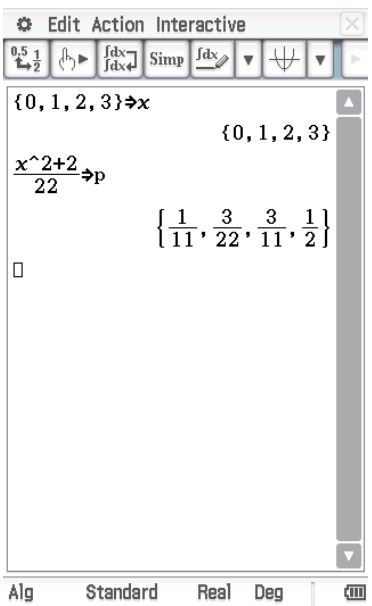
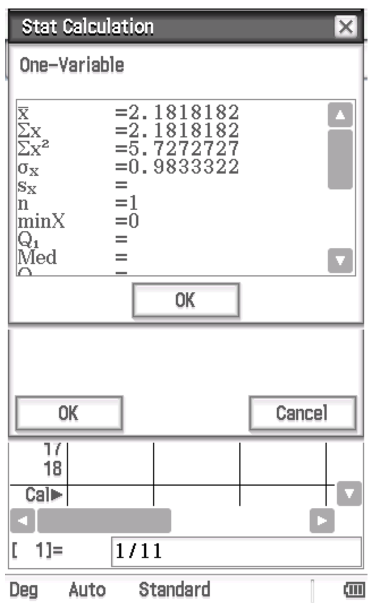




a $P(Z = z) = \frac{z^2 + 2}{22}$ when $x = 0, 1, 2, 3$.

z	0	1	2	3
$p(Z = z)$	$\frac{1}{11}$	$\frac{3}{22}$	$\frac{3}{11}$	$\frac{1}{2}$

b $E(Z) = 0 \times \frac{1}{11} + 1 \times \frac{3}{22} + 2 \times \frac{3}{11} + 3 \times \frac{1}{2}$

$= \frac{24}{11} = 2.\overline{18}$

$= 2.182\,(3\,\text{dp})$

c $E(Z^2) = 0^2 \times \frac{1}{11} + 1^2 \times \frac{3}{22} + 2^2 \times \frac{3}{11} + 3^2 \times \frac{1}{2}$

$= \frac{63}{11} = 5.\overline{72}$

$\sigma_z^2 = E(Z^2) - \mu^2$

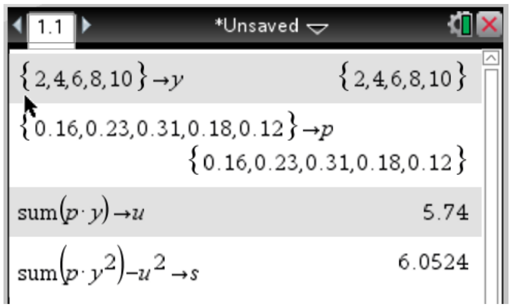
$= 5.\overline{72} - 2.\overline{18}^2 = 0.96694$

$\sigma_z = 0.983$ (3 dp)

$\mu = 2.182$ (3 dp) and $\sigma_z = 0.983$ (3 dp)

11

TI-Nspire CAS



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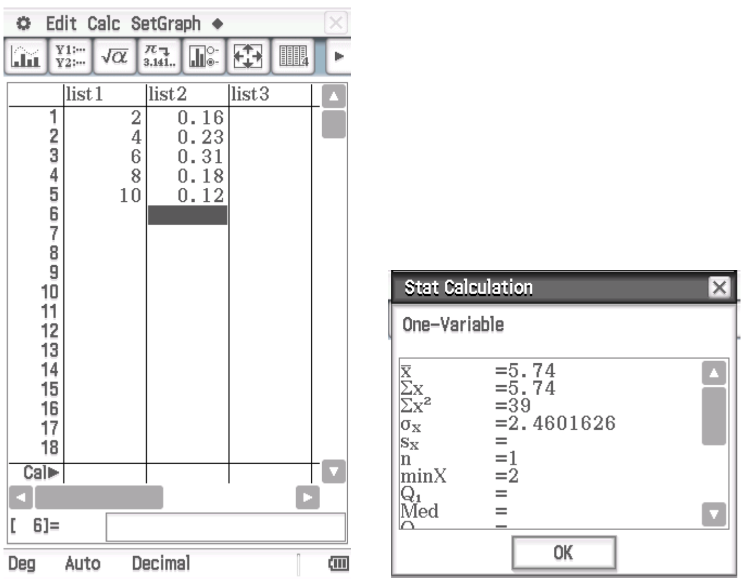


$E(Y) = 2 \times 0.16 + 4 \times 0.23 + 6 \times 0.31 + 8 \times 0.18 + 10 \times 0.12$

$= 5.74$

$E(Y^2) = 2^2 \times 0.16 + 4^2 \times 0.23 + 6^2 \times 0.31 + 8^2 \times 0.18 + 10^2 \times 0.12$

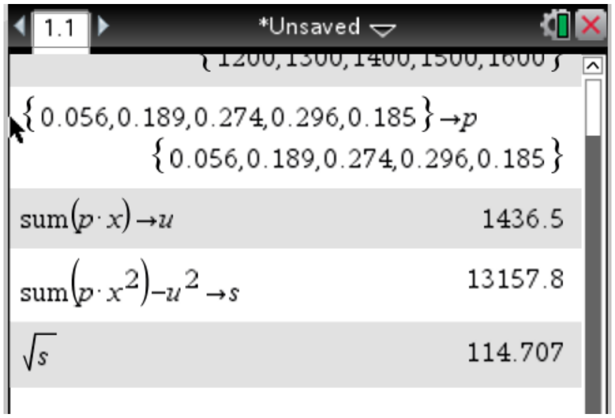
$= 39$

$\sigma_y^2 = E(Y^2) - \mu^2$

$= 39 - 5.74^2 = 6.05$

12 a

TI-Nspire CAS



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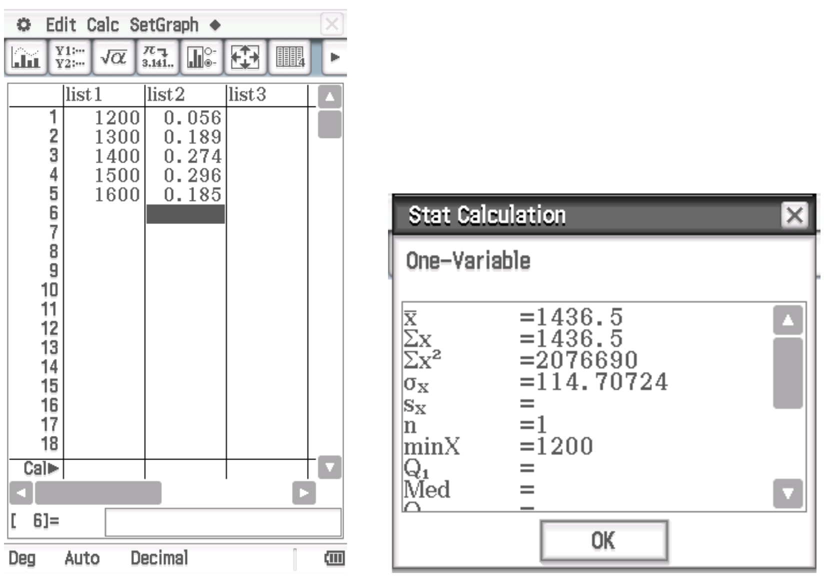


$E(x) = 1200 \times 0.056 + 1300 \times 0.189 + 1400 \times 0.274 + 1500 \times 0.296$

$+ 1600 \times 0.185$

≈ 1437

$E(x^2) = 1200^2 \times 0.056 + 1300^2 \times 0.189 + 1400^2 \times 0.274 + 1500^2 \times 0.296$

$+ 1600^2 \times 0.185$

$= 2\,076\,690$

$\sigma_x^2 = E(x^2) - \mu^2$

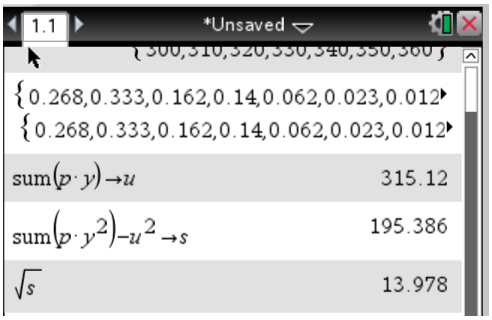
$= 2\,076\,690 - 1436.5^2$

$= 13157.75$

$\sigma_x = 114.7$

b

TI-Nspire CAS



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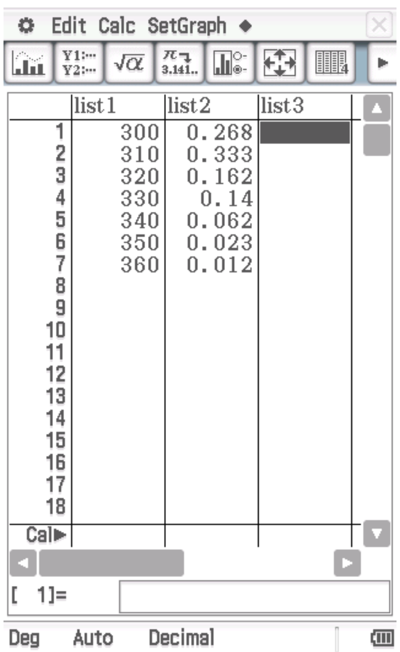

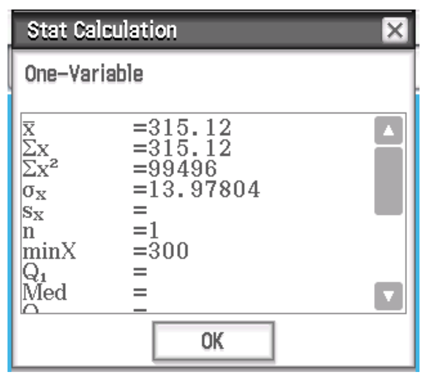

$E(y) = 300 \times 0.268 + 310 \times 0.333 + 320 \times 0.162 + 330 \times 0.14 + 340 \times 0.062$

$+ 350 \times 0.023 + 360 \times 0.012$

$= 315.12$

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$$E(y^2) = 300^2 \times 0.268 + 310^2 \times 0.333 + 320^2 \times 0.162 + 330^2 \times 0.14 + 340^2 \times 0.062$$

$$+ 350^2 \times 0.023 + 360^2 \times 0.012$$

$$= 99\,496$$

$$\sigma_y^2 = E(y^2) - \mu^2$$

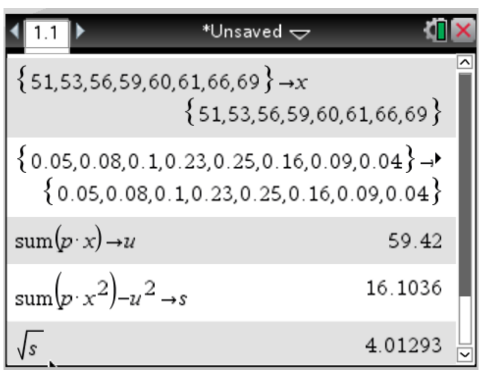
$$= 99\,496 - 315.12^2$$

$$= 195.39$$

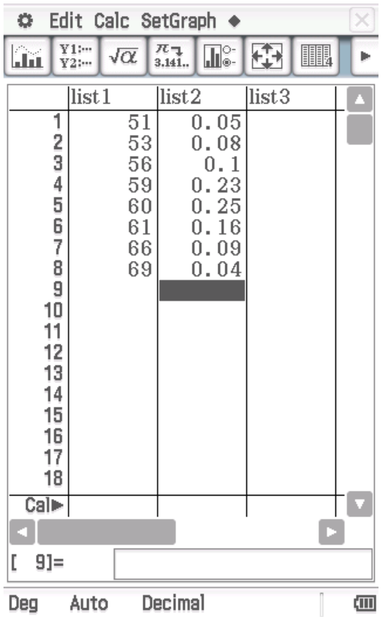
$$\sigma_y = 13.98 \approx 14$$

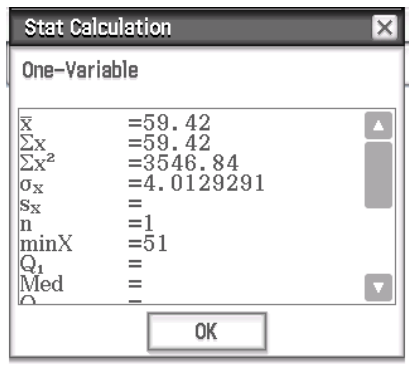
13

TI-Nspire



ClassPad





a $E(X) = 51 \times 0.05 + 53 \times 0.08 + 56 \times 0.1 + 59 \times 0.23 + 60 \times 0.25$

$+ 61 \times 0.16 + 66 \times 0.09 + 69 \times 0.04$

$= 59.42$

b $E(X^2) = 51^2 \times 0.05 + 53^2 \times 0.08 + 56^2 \times 0.1 + 59^2 \times 0.23 + 60^2 \times 0.25$

$+ 61^2 \times 0.16 + 66^2 \times 0.09 + 69^2 \times 0.04$

$= 3546.84$

$\sigma_x^2 = E(X^2) - \mu^2$

$= 3546.84 - 59.42^2$

$= 16.1$

c $\sigma_x = 4.01$

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Reasoning and communication

14 $E(X) = 1 \times 0.2 + 3 \times 0.4 + k \times 0.1 + 6 \times 0.3$

$= 3.2 + 0.1k$

$E(X^2) = 1^2 \times 0.2 + 3^2 \times 0.4 + k^2 \times 0.1 + 6^2 \times 0.3$

$= 14.6 + 0.1k^2$

$\sigma_x^2 = E(X^2) - \mu^2$

$= 14.6 + 0.1k^2 - (3.2 + 0.1k)^2$

But $\sigma_x^2 = 3.41$

so $3.41 = 14.6 + 0.1k^2 - (3.2 + 0.1k)^2$

$k = 5$ (k is integer)

15 $E(Y) = 1 \times 0.1 + k \times 0.3 + 7 \times 0.4 + 11 \times 0.2$

$= 5.1 + 0.3k$

$E(Y^2) = 1^2 \times 0.1 + k^2 \times 0.3 + 7^2 \times 0.4 + 11^2 \times 0.2$

$= 43.9 + 0.3\text{k}^2$

$\sigma_y^2 = E(Y^2) - \mu^2$

$= 43.9 + 0.3k^2 - (5.1 + 0.3k)^2$

But $\sigma_y^2 = 10.6$

so $10.6 = 43.9 + 0.3k^2 - (5.1 + 0.3k)^2$

$k = 3$ (k is integer)

 ISBN 9780170254663

16 Faces of two dice — Sum of the faces (S)

Faces of two dice	Sum of the faces (S)
(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)	(2) (3) (4) (5) (6) (7)
(2, 1) (2, 2) (2, 3) (2, 4) (2, 5) (2, 6)	(3) (4) (5) (6) (7) (8)
(3, 1) (3, 2) (3, 3) (3, 4) (3, 5) (3, 6)	(4) (5) (6) (7) (8) (9)
(4, 1) (4, 2) (4, 3) (4, 4) (4, 5) (4, 6)	(5) (6) (7) (8) (9) (10)
(5, 1) (5, 2) (5, 3) (5, 4) (5, 5) (5, 6)	(6) (7) (8) (9) (10) (11)
(6, 1) (6, 2) (6, 3) (6, 4) (6, 5) (5, 6)	(7) (8) (9) (10) (11) (12)

x	2	3	4	5	6	7	8	9	10	11	12
$p(x)$	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

a $$E(S) = 2 \times \frac{1}{36} + 3 \times \frac{1}{18} + 4 \times \frac{1}{12} + 5 \times \frac{1}{9} + 6 \times \frac{5}{36} + 7 \times \frac{1}{6} + 8 \times \frac{5}{36} + 9 \times \frac{1}{9} + 10 \times \frac{1}{12} + 11 \times \frac{1}{18} + 12 \times \frac{1}{36}$$

$$= 7$$

b $$E(S^2) = 2^2 \times \frac{1}{36} + 3^2 \times \frac{1}{18} + 4^2 \times \frac{1}{12} + 5^2 \times \frac{1}{9} + 6^2 \times \frac{5}{36} + 7^2 \times \frac{1}{6} + 8^2 \times \frac{5}{36} + 9^2 \times \frac{1}{9} + 10^2 \times \frac{1}{12} + 11^2 \times \frac{1}{18} + 12^2 \times \frac{1}{36}$$

$$= 54.83$$

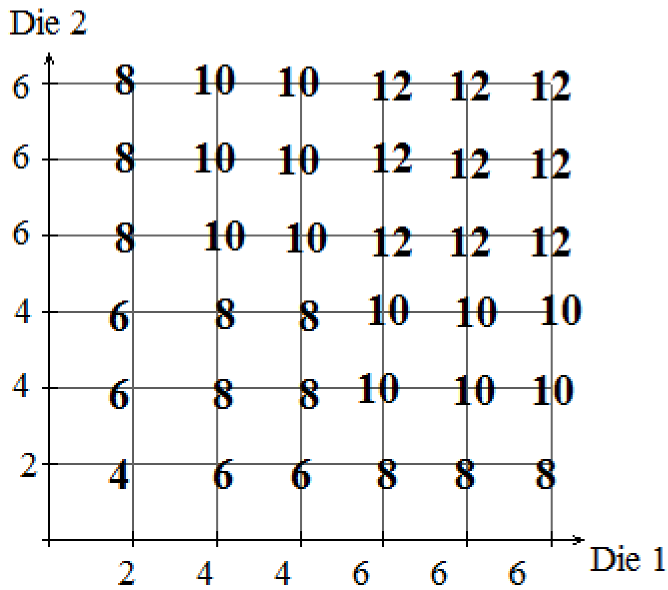
$$\sigma_s^2 = E(S^2) - \mu^2$$

$$= 54.83 - 7^2$$

$$= 5.83$$

c $\sigma_s = 2.42$

17 **a**



t	4	6	8	10	12
p(*t*)	$\frac{1}{36}$	$\frac{1}{9}$	$\frac{5}{18}$	$\frac{1}{3}$	$\frac{1}{4}$

b $E(T) = 4 \times \frac{1}{36} + 6 \times \frac{1}{9} + 8 \times \frac{5}{18} + 10 \times \frac{1}{3} + 12 \times \frac{1}{4}$

$= 9.33$

$E(T^2) = 4^2 \times \frac{1}{36} + 6^2 \times \frac{1}{9} + 8^2 \times \frac{5}{18} + 10^2 \times \frac{1}{3} + 12^2 \times \frac{1}{4}$

$= 91.55$

$\sigma_t^2 = E(T^2) - \mu^2$

$= 91.55 - 9.33^2$

$= 4.44$

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18 The outcomes | X (The sum) | Y (The smaller)

The outcomes	X (The sum)	Y (The smaller)
(1, 1) (1, 2) (1, 3)	(2) (3) (4)	(1) (1) (1)
(2, 1) (2, 2) (2, 3)	(3) (4) (5)	(1) (2) (2)
(3, 1) (3, 2) (3, 3)	(4) (5) (6)	(1) (2) (3)

a

X	2	3	4	5	6
$P(X = x)$	$\frac{1}{9}$	$\frac{2}{9}$	$\frac{3}{9}$	$\frac{2}{9}$	$\frac{1}{9}$

Y	1	2	3
$P(Y = y)$	$\frac{5}{9}$	$\frac{3}{9}$	$\frac{1}{9}$

b $E(X) = 2 \times \frac{1}{9} + 3 \times \frac{2}{9} + 4 \times \frac{3}{9} + 5 \times \frac{2}{9} + 6 \times \frac{1}{9}$

$= 4$

$E(X^2) = 2^2 \times \frac{1}{9} + 3^2 \times \frac{2}{9} + 4^2 \times \frac{3}{9} + 5^2 \times \frac{2}{9} + 6^2 \times \frac{1}{9}$

$= 17.33\ldots$

$\sigma_x^2 = E(X^2) - \mu^2$

$= 17.33 - 4^2$

$= 1.33\ldots$

c $E(Y) = 1 \times \frac{5}{9} + 2 \times \frac{3}{9} + 3 \times \frac{1}{9}$

$= 1.56\ldots$

$E(Y^2) = 1^2 \times \frac{5}{9} + 2^2 \times \frac{3}{9} + 3^2 \times \frac{1}{9}$

$= 2.88\ldots$

$\sigma_y^2 = E(Y^2) - \mu^2$

$= 2.89 - 1.55^2$

$= 0.469\ldots$

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Exercise 2.08 Applications of discrete random variables

Reasoning and communication

1 **a**

	3 dice same	**2 dice same**	**No dice same**
***X* (winnings)**	$5	0	–$5
***P*(*X* = *x*)**	$\frac{6}{216}$	$\frac{90}{216}$	$\frac{120}{216}$

$$E(X) = 5 \times \frac{6}{216} + 0 \times \frac{90}{216} + (-5) \times \frac{120}{216} = -\$\frac{570}{216}$$

$= –\$2.64$

b No, the game is not fair as you are averaging a loss.

2

	Three 4s **444**	**Two 4s** **44***	**One 4** **4****	**No fours** *****
***X* (winnings)**	$3	$2	$1	–$1
***P*(*X* = *x*)**	$\frac{1}{216}$	$\frac{15}{216}$	$\frac{75}{216}$	$\frac{125}{216}$

$$E(X) = 3 \times \frac{1}{216} + 2 \times \frac{15}{216} + 1 \times \frac{75}{216} + (-1) \times \frac{125}{216} = -\$\frac{17}{216}$$

$= –\$0.08$

The house percentage $= \frac{0.07878}{1} \times 100\% = 7.9\%$

3

x	\$29 9350	–\$650
***P*(*X* = *x*)**	0.00 212	0.997 88

$E(X)$ = \$29 9350 × 0.002 12 + (–\$650) × 0.997 88

= –\$14

The expected value for each policy for the insurance company is \$14.

4 $E(X) = 0 \times 0.1 + 1 \times 0.15 + 2 \times 0.15 + 3 \times 0.2 + 4 \times 0.4$

$= 2.65$

5 **a** $E(X) = 0 \times 0.95 + 1 \times 0.03 + 2 \times 0.015 + 3 \times 0.003 + 4 \times 0.0015 + 5 \times 0.0005$

$= 0.0775$

b 0

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6 **a** P(fewer than 4 persons are in any given household) = 0.269 + 0.334 + 0.162

$= 0.765$

b $E(X) = 1 \times 0.269 + 2 \times 0.334 + 3 \times 0.162 + 4 \times 0.140 + 5 \times 0.062 + 6 \times 0.023$

$+ 7 \times 0.012$

$= 2.51$

c $E(X^2) = 1^2 \times 0.269 + 2^2 \times 0.334 + 3^2 \times 0.162 + 4^2 \times 0.140 + 5^2 \times 0.062$

$+ 6^2 \times 0.023 + 7^2 \times 0.012$

$= 8.269$

$\sigma_x^2 = E(X^2) - \mu^2$

$= 8.269 - 2.51^2$

$= 1.969$

$\sigma_x \approx 1.4$

7 $E(X) = -1 \times 0.3 + 0 \times 0.4 + 3 \times 0.2 + 5 \times 0.1$

$= 0.8$

Favours the player by 80 cents a game.

8 **a**

Outcome ($)	−2	−1	2	4
Probability	0.3	0.4	0.2	0.1

$E(X) = (-2) \times 0.3 + (-1) \times 0.4 + 2 \times 0.2 + 4 \times 0.1$

$= -0.2$

Favours the house by 20 cents a game.

b

Outcome ($)	−4	−2	4	8
Probability	0.3	0.4	0.2	0.1

$E(X) = (-4) \times 0.3 + (-2) \times 0.4 + 4 \times 0.2 + 8 \times 0.1$

$= -0.4$

Favours the house by 40 cents a game instead of 20 cents a game.

9

Outcome ($)	$20 000 – $10 = $19 990	$500 –$10 = $490	–$10
Probability	0.0001	0.0020	0.9979

$E(X)$ = $19 990 × 0.0001 + $490 × 0.0020 + (–$10) × 0.9979

= –$7

Each person loses an average of $7, NOT a good investment.

10

Outcome ($)	Blue $0	Black $3	Green $5	Other –$5
Probability	0.25	0.196	0.196	0.357

Area of black: Assume square has a side of one unit.

Radius is 0.25. $A_O = \pi r^2 = 0.196\ 3495 \approx 0.196$

$E(X)$ = $0 × 0.25 + $3 × 0.196 + $5 × 0.196 + (–$5) × 0.357

= –$0.217

On average, you would lose money if you play, about 22 cents per game.

11

x	\$72 500	–\$2500
***P*(*X* = *x*)**	0.017	0.983

$E(X)$ = \$72 500 × 0.017 + (–\$2500) × 0.983

= –\$1225

The insurance company will make \$1225 on each policy.

12

x	\$(50 000 – *a*)	–\$*a*
***P*(*X* = *x*)**	0.009	0.991

$E(X)$ = \$(50 000 – a) × 0.009 + (–\$$a$) × 0.991

Given $E(X)$ = –\$225

∴ –225 = (50 000 – a) × 0.009 + (–a) × 0.991

$a = 675$

To achieve the desired profit, the insurance company needs to charge a 38-year-old male a \$675 premium.

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13

x	\$970	–\$30
***P*(*X* = *x*)**	0.005	0.995

a $E(X) = \$970 \times 0.005 + (-\$30) \times 0.995$

$= -\$25$

The expected gain for the insurer for each thousand dollars of coverage is \$25.

b \$3000, as $3 \times \$25 = \$75$, which covers the costs.

14 **a** $E(X) = 0 \times 0.90 + 1 \times 0.06 + 2 \times 0.02 + 3 \times 0.08 + 4 \times 0.006 + 5 \times 0.006$

$= 0.178$

b Pay per day = $\$300 + 0.178 \times \$200 = \$335.60$

c \$300

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15 Faces of two dice Sum of the faces S

Faces of two dice						Sum of the faces S					
(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)	(2)	(3)	(4)	(5)	(6)	(7)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)	(3)	(4)	(5)	(6)	(7)	(8)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)	(4)	(5)	(6)	(7)	(8)	(9)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)	(5)	(6)	(7)	(8)	(9)	(10)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)	(6)	(7)	(8)	(9)	(10)	(11)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(5, 6)	(7)	(8)	(9)	(10)	(11)	(12)

x	7 or 11 –$3	Double $6	Otherwise $0
$p(x)$	$\frac{8}{36}$	$\frac{6}{36}$	$\frac{22}{36}$

a $E(X) = -3 \times \frac{8}{36} + 6 \times \frac{6}{36} + 0 \times \frac{22}{36}$

$= \$0.33$

b 33 cents per game

16 **a**

x	$10 000	$5 000	–$1,000
$p(x)$	0.2	0.2	0.6

$E(X) = \$10\,000 \times 0.2 + \$5000 \times 0.2 + (-\$1000) \times 0.6$

$= \$2400$

b –$1000

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17

	Probability
AA	0.5^2
BB	0.5^2
BAA	0.5^3
ABB	0.5^3
ABAA	0.5^4
BABB	0.5^4
ABABA	0.5^5
BABAA	0.5^5
ABABB	0.5^5
BABAB	0.5^5

x	2 games	3 games	4 games	5 games
***p*(*x*)**	0.5	0.25	0.125	0.125

$E(X) = 2 \times 0.5 + 3 \times 0.25 + 4 \times 0.125 + 5 \times 0.125$

$= 2.875$

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18 You would expect the standard deviation for the first exam to be more than in the second.

19 $E(X) = 100 \times 0.15 + 250 \times 0.35 + 300 \times 0.25 + 350 \times 0.15 + 400 \times 0.1$

$= \$270$

The expected return is $270 000.

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Chapter 2 Review

Multiple choice

1 D because $P(C \cup D) = P(C) + P(D) - P(C \cap D)$

$= 0.75 + 0.36 - 0.29$

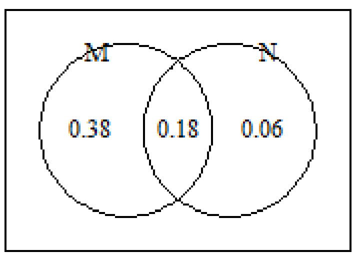
$= 0.82$

2 E $P(M|N) = ?$ $P(M \cup N) = P(M) + P(N) - P(M \cap N)$

$P(M \cup N) = 0.56 + 0.24 - 0.18$

$P(M \cup N) = 0.62$

$P(M|N) = \dfrac{0.18}{0.24} = 0.75$



3 E $P(0) = \dfrac{1}{8}$, $P(1) = \dfrac{3}{8}$, $P(2) = \dfrac{3}{8}$, $P(3) = \dfrac{1}{8}$

as HHH, HHT, HTH,THH, TTH,THT,HTT, TTT

Ratios 1 : 3 : 3: 1 out of 8

4 A As the value must be between 0 and 1 inclusive.

5 B Only specific values as there are a discrete number of values, each of which has a definite probability.

6 A The total of $p(x)$ is 1 and they are all positive.

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7 A 4 With 8 possible successes, $x \in \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$ but sample size is 4, so max is 4.

8 C $P(X = 4) = \dfrac{\binom{10}{4}\binom{22}{8}}{\binom{32}{12}}$ $\Rightarrow$ breakdown of 32 into 10 and 22.

Traditionally the hypergeometric distribution is defined as

$$P(X = x) = \frac{\binom{k}{x}\binom{N-k}{n-x}}{\binom{N}{n}}$$

9 B $E(X) = 1 \times 0.2 + 2 \times 0.3 + 3 \times 0.4 + 4 \times 0.1$

$= 2.4$

10 B $E(X^2) = 1^2 \times 0.2 + 2^2 \times 0.3 + 3^2 \times 0.4 + 4^2 \times 0.1$

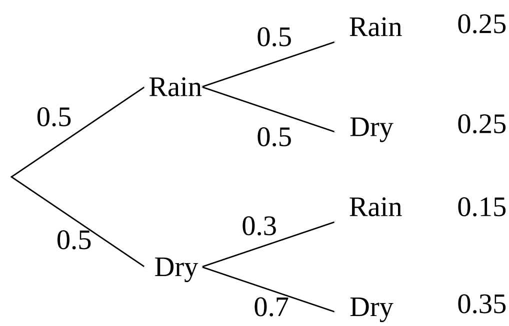
$= 6.6$

$\sigma_x^2 = E(X^2) - \mu^2$

$= 6.6 - 2.4^2$

$= 0.84$

11 C 0.9



Let X be the number of rainy days $x \in \{0, 1, 2\}$

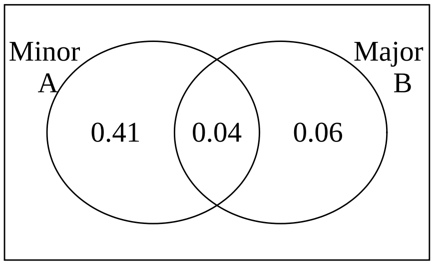
x	0	1	2
***p*(*x*)**	0.35	0.4	0.25

$E(X) = 0 \times 0.35 + 1 \times 0.4 + 2 \times 0.25$

$= 0.9$

Short answer

12



$P(\text{A}) = 0.45$, $P(\text{B}) = 0.1$, $P(\text{A} \cap \text{B}) = 0.04$

a P(not a minor repair but will need a major repair) = $P(\text{B} \mid \overline{\text{A}}) = 0.06$

b P(not need either a minor or major repair) = 1 – 0.45 – 0.06 = 0.49

c P(major repair required, given car requires a minor repair)

$$= P(\text{B} \mid \text{A}) = \frac{0.04}{0.45} = 0.089$$

13 **a** discrete

b continuous

c discrete

d continuous

e discrete

f discrete

g continuous

h discrete

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14 **a** Could represent a probability distribution as the total is equal to one.

b Could represent a probability distribution as the total is equal to one.

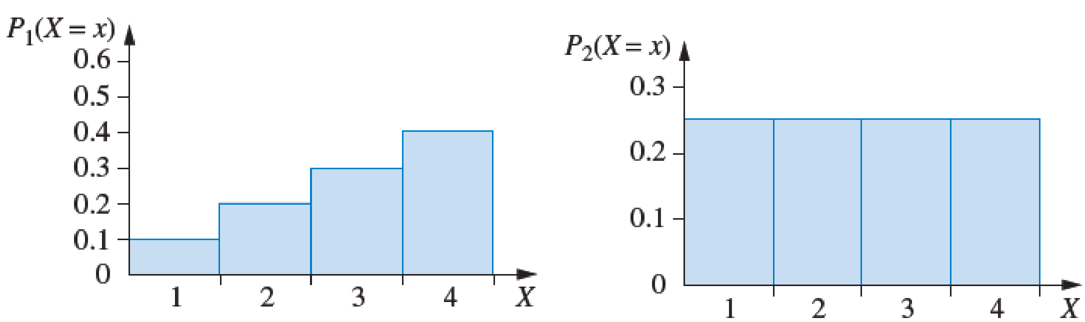
c Could not represent a probability distribution as probabilities cannot equal a negative number.

15 **a**

X	1	2	3	4
$p_1(X=x)$	0.1	0.2	0.3	0.4

X	1	2	3	4
$P_2(X=x)$	0.25	0.25	0.25	0.25

b



c $\sum p(x) = 1$ and $0 \le p(x) \le 1$, for each distribution.

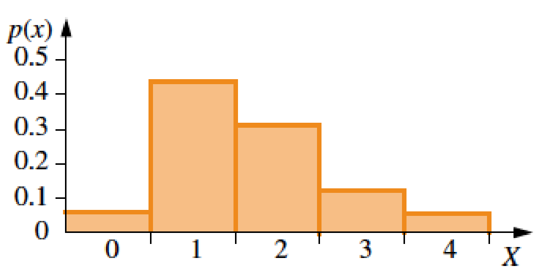
d The second distribution is uniform because the probabilities are identical.

16 a

	Consecutive heads
HHHH	4
HHHT	3
HHTH	2
HTHH	2
THHH	3
HHTT	2
HTHT	1
HTTH	1
THHT	2
THTH	1
TTHH	2
TTTH	1
TTHT	1
THTT	1
HTTT	1
TTTT	0

X	0	1	2	3	4
$P(X = x)$	$\frac{1}{16}$	$\frac{7}{16}$	$\frac{5}{16}$	$\frac{1}{8}$	$\frac{1}{16}$

b



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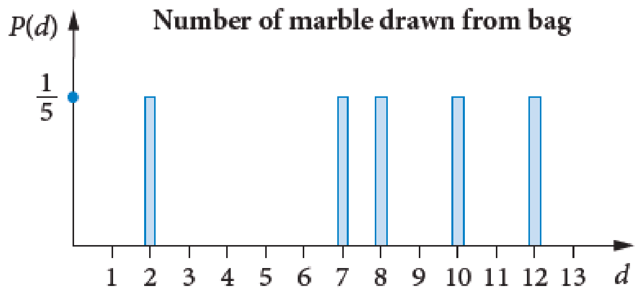
17 **a**



b $P(\text{odd}) = P(7) = \frac{1}{5}$

c The distribution is uniform as all the probabilities are the same.

18 **a**

t	11	12	13	14	15	16	17	18	19	20
f	2	3	4	6	1	3	0	1	1	4
$P(T = t)$	0.08	0.12	0.16	0.24	0.04	0.12	0	0.04	0.04	0.16

b 3 out of 25, i.e. 0.12

c 10 out of 25, i.e. 0.4

19 **a** $N = 40$, $n = 5$, $k = 7$, $x \in \{0, 1, 2, 3, 4, 5\}$

b $N = 5000$, $n = 10$, $k = 1000$, $x \in \{0, 1, 2, \ldots, 10\}$

c $N = 100$, $n = 15$, $k = 12$, $x \in \{0, 1, 2, \ldots, 12\}$

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20 **a** $N = 40$, $n = 10$, $k = 12$, $x = 5$, $P(x=5) = \dfrac{{}^{12}C_5 \, {}^{28}C_5}{{}^{40}C_{10}} = 0.0918$

b $N = 65$, $n = 12$, $k = 25$, $x = 4$, $P(x=4) = \dfrac{{}^{25}C_4 \, {}^{40}C_8}{{}^{65}C_{12}} = 0.2415$

c $N = 250$, $n = 60$, $k = 30$, $x = 15$, $P(x=15) = \dfrac{{}^{30}C_{15} \, {}^{220}C_{45}}{{}^{250}C_{60}} = 0.000\,656$

21 **a**

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Differences

0	1	2	3	4	5
1	0	1	2	3	4
2	1	0	1	2	3
3	2	1	0	1	2
4	3	2	1	0	1
5	4	3	2	1	0

d	0	1	2	3	4	5
***P* (*D* = *d*)**	$\frac{1}{6}$	$\frac{5}{18}$	$\frac{2}{9}$	$\frac{1}{6}$	$\frac{1}{9}$	$\frac{1}{18}$

b $E(D) = 0 \times \dfrac{1}{6} + 1 \times \dfrac{5}{18} + 2 \times \dfrac{2}{9} + 3 \times \dfrac{1}{6} + 4 \times \dfrac{1}{9} + 5 \times \dfrac{1}{18}$

$= 1.944$

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t	1	2	3	4	5	6	7	8	9	10
frequency	2	3	4	2	8	4	3	4	1	2
$P(T = t)$	$\frac{2}{33}$	$\frac{3}{33}$	$\frac{4}{33}$	$\frac{2}{33}$	$\frac{8}{33}$	$\frac{4}{33}$	$\frac{3}{33}$	$\frac{4}{33}$	$\frac{1}{33}$	$\frac{2}{33}$

$$E(T) = 1 \times \frac{2}{33} + 2 \times \frac{3}{33} + 3 \times \frac{4}{33} + 4 \times \frac{2}{33} + 5 \times \frac{8}{33} + 6 \times \frac{4}{33} + 7 \times \frac{3}{33} + 8 \times \frac{4}{33}$$

$$+ 9 \times \frac{1}{33} + 10 \times \frac{2}{33}$$

$$= 5.273$$

$$E(T^2) = 1^2 \times \frac{2}{33} + 2^2 \times \frac{3}{33} + 3^2 \times \frac{4}{33} + 4^2 \times \frac{2}{33} + 5^2 \times \frac{8}{33} + 6^2 \times \frac{4}{33} + 7^2 \times \frac{3}{33}$$

$$+ 8^2 \times \frac{4}{33} + 9^2 \times \frac{1}{33} + 10^2 \times \frac{2}{33}$$

$$= 33.636$$

$$\sigma_t^2 = E(T^2) - \mu^2$$

$$= 33.636 - 5.273^2$$

$$= 5.83$$

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23 $E(X) = -2 \times 0.25 + 0 \times 0.125 + 2 \times 0.5 + 4 \times 0.125$

$= 1$

$E(X^2) = (-2)^2 \times 0.25 + 0^2 \times 0.125 + 2^2 \times 0.5 + 4^2 \times 0.125$

$= 5$

$\sigma_x^2 = E(X^2) - \mu^2$

$= 5 - 1^2$

$= 4$

24 **a** $E(Y) = 200 \times 0.05 + 300 \times 0.15 + 400 \times 0.35 + 500 \times 0.25 + 600 \times 0.15$

$+ 800 \times 0.05$

$= 450$

b $E(Y^2) = 200^2 \times 0.05 + 300^2 \times 0.15 + 400^2 \times 0.35 + 500^2 \times 0.25 + 600^2 \times 0.15$

$+ 800^2 \times 0.05$

$= 220\,000$

$\sigma_y^2 = E(Y^2) - \mu^2$

$= 220\,000 - 450^2$

$= 17\,500$

c $\sigma_y = 132.3$

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Application

25 **a**

	Consecutive heads	Probability
HHH	3	$\left(\frac{3}{4}\right)^3 = \frac{27}{64}$
HHT	2	$\left(\frac{3}{4}\right)^2 \times \frac{1}{4} = \frac{9}{64}$
HTH	2	$\left(\frac{3}{4}\right)^2 \times \frac{1}{4} = \frac{9}{64}$
THH	2	$\left(\frac{3}{4}\right)^2 \times \frac{1}{4} = \frac{9}{64}$
HTT	1	$\frac{3}{4} \times \left(\frac{1}{4}\right)^2 = \frac{3}{64}$
THT	1	$\frac{3}{4} \times \left(\frac{1}{4}\right)^2 = \frac{3}{64}$
TTH	1	$\frac{3}{4} \times \left(\frac{1}{4}\right)^2 = \frac{3}{64}$
TTT	0	$\left(\frac{1}{4}\right)^3 = \frac{1}{64}$

X	0	1	2	3
$P(X = x)$	$\frac{1}{64}$	$\frac{18}{64} = \frac{9}{32}$	$\frac{18}{64} = \frac{9}{32}$	$\frac{27}{64}$

b $E(X) = 0 \times \frac{1}{64} + 1 \times \frac{18}{64} + 2 \times \frac{18}{64} + 3 \times \frac{27}{64}$

$= 2.1$

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(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

Product (D)

1	2	3	4	5	6
2	4	6	8	10	12
3	6	9	12	15	18
4	8	12	16	20	24
5	10	15	20	25	30
6	12	18	24	30	36

d	1	2	3	4	5	6	8	9	10	12	15	16	18	20	24	25	30	36
frequency	1	2	2	3	2	4	2	1	2	4	2	1	2	2	2	1	2	1
***P*(*D* = *d*)**	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{4}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{2}{36}$	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

$$E(D) = 1 \times \frac{1}{36} + 2 \times \frac{2}{36} + 3 \times \frac{2}{36} + 4 \times \frac{3}{36} + 5 \times \frac{2}{36} + 6 \times \frac{4}{36} + 8 \times \frac{2}{36} + 9 \times \frac{1}{36}$$

$$+ 10 \times \frac{2}{36} + 12 \times \frac{4}{36} + 15 \times \frac{2}{36} + 16 \times \frac{1}{36} + 18 \times \frac{2}{36} + 20 \times \frac{2}{36}$$

$$+ 24 \times \frac{2}{36} + 25 \times \frac{1}{36} + 30 \times \frac{2}{36} + 36 \times \frac{1}{36}$$

$$= 12.25$$

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$$E(D^2) = 1^2 \times \frac{1}{36} + 2^2 \times \frac{2}{36} + 3^2 \times \frac{2}{36} + 4^2 \times \frac{3}{36} + 5^2 \times \frac{2}{36} + 6^2 \times \frac{4}{36} + 8^2 \times \frac{2}{36}$$

$$+ 9^2 \times \frac{1}{36} + 10^2 \times \frac{2}{36} + 12^2 \times \frac{4}{36} + 15^2 \times \frac{2}{36} + 16^2 \times \frac{1}{36} + 18^2 \times \frac{2}{36}$$

$$+ 20^2 \times \frac{2}{36} + 24^2 \times \frac{2}{36} + 25^2 \times \frac{1}{36} + 30^2 \times \frac{2}{36} + 36^2 \times \frac{1}{36}$$

$$= 238.83$$

$$\sigma_d^2 = E(D^2) - \mu^2$$

$$= 238.83 - 12.25^2$$

$$= 79.97$$

The distribution is not uniform as the probabilities are different.

27 80 = 14 + 66

$N = 80$, $k = 14$, $n = 15$

a P(four will be defective) = $P\left(x = 4\right) = \frac{^{14}C_4 \times {}^{66}C_{11}}{^{80}C_{15}} = 0.1620$

b P(at least three will be defective)

$= P(3) + P(4) + \ldots$

$= 1 - P(0) - P(1) - P(2)$

$$= 1 - \frac{^{66}C_{15}}{^{80}C_{15}} - \frac{^{14}C_1 \times {}^{66}C_{14}}{^{80}C_{15}} - \frac{^{14}C_2 \times {}^{66}C_{13}}{^{80}C_{15}}$$

$= 1 - 0.0404\ldots - 0.1633\ldots - 0.2804\ldots$

$= 0.5158\ldots$

28

X	\$99 999	\$24 999	\$999	\$99	\$9	\$1	–\$1
***P*(*X* = *x*)**	$\frac{1}{10^7}$	$\frac{2}{10^7}$	$\frac{25}{10^7}$	$\frac{25000}{10^7}$	$\frac{25000}{10^7}$	0.12	0.875

a 0.125 000 28

b $E(X) = \$99\,999 \times \frac{1}{10^7} + \$24\,999 \times \frac{2}{10^7} + \$999 \times \frac{25}{10^7}$

$+ \$99 \times \frac{25000}{10^7} + \$9 \times \frac{25000}{10^7} + \$1 \times 0.12 - \$1 \times 0.875$

$= -0.4675$

Payout is \$1 – 0.4675 = 53 cents per ticket.

c House percentage = 0.4675 × 100% ≈ 47%

29 Two six-sided dice — Sum of faces (S)

Two six-sided dice						Sum of faces (S)					
(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)	(2)	(3)	(4)	(5)	(6)	(7)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)	(3)	(4)	(5)	(6)	(7)	(8)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)	(4)	(5)	(6)	(7)	(8)	(9)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)	(5)	(6)	(7)	(8)	(9)	(10)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)	(6)	(7)	(8)	(9)	(10)	(11)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)	(7)	(8)	(9)	(10)	(11)	(12)

s	Less than 6 or greater than 9 \$10	Otherwise –\$5
***P* (*S* = *s*)**	$\frac{16}{36}$	$\frac{20}{36}$

a $E(S) = \$10 \times \frac{16}{36} + (-\$5) \times \frac{20}{36}$

$= \$1.67$

b No. More chance of winning than losing.

c Change the sums that are included so winning and losing were equal.

E.g. less than 6 or an 8, 11 or 12; more than 7 or a 4

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30

X	\$10 000	\$5 000	–\$1000
***P*(*X* = *x*)**	0.2	0.2	0.6

a $E(X)$ = \$10 000 × 0.2 + \$5 000 × 0.2+ (–\$1000) × 0.6

= \$2400

b The most likely value of X is –\$1000.

31 $P(0W) = \dfrac{{}^{17}C_3 \times {}^{3}C_0}{{}^{20}C_3} = 0.5965$

$P(1W) = \dfrac{{}^{17}C_2 \times {}^{3}C_1}{{}^{20}C_3} = 0.3579$

$P(2W) = \dfrac{{}^{17}C_1 \times {}^{3}C_2}{{}^{20}C_3} = 0.0447$

$P(3W) = \dfrac{{}^{17}C_0 \times {}^{3}C_3}{{}^{20}C_3} = 0.00008$

X	3W \$4.50	2W \$1.50	1W \$0	0W –\$0.50
***P*(*X* = *x*)**	0.000 08	0.0447	0.3579	0.5965

$E(X)$ = \$4.50 × 0.000 08 + \$1.50 × 0.0447 + (–\$0.50) × 0.5965

= –\$0.23

The player should expect to lose 23 cents on average per game.

32 Sales demand:

$E(X) = 5000 \times 0.3 + 6000 \times 0.6 + 8000 \times 0.1$

$= 5900$

Variable costs:

$E(X) = \$3 \times 0.1 + \$3.50 \times 0.3 + \$4 \times 0.5 + \$4.50 \times 0.1$

$= \$3.80$

Sale price = \$6 – \$3.80 = \$2.20

Revenue – costs = 5900 × \$2.2 – \$8000 = \$4980

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